A More General Framework to Analyze Whether Voluntary Disclosure is Insufficient or Excessive*

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Abstract

I analyze if the excessive quality disclosure finding of the "classical literature" extends to environments in which consumers have a downward-sloping demand. While the answer is affirmative, there are at least two situations under which disclosure is socially insufficient: (1) when there are quality levels that are too low to generate any positive demand; and (2) when the prior beliefs place sufficiently higher weight on lower qualities. In both cases, non-disclosure by the seller leads to a severe reduction in the perceived quality, thereby significantly lowering the demand and the quantity consumed.

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1 Introduction

When potential buyers of a product are unable to observe the product quality prior to purchase but know that the seller is informed about it, they may anticipate that this information will be communicated, especially in favorable situations that benefit the seller. In fact, if the seller has access to a verifiable and costless means of disclosing information, then quality will be fully revealed in all equilibria. This is the so-called 'information unraveling' result, which was first established by Grossman (1981), Grossman and Hart (1980), and Milgrom (1981).[1] In the words of Grossman and Hart (1980):

> The buyer must use the simple logic that the seller tries to be as optimistic as possible about his product subject to the constraint that he not lie.

When disclosure involves a strictly positive cost, on the other hand, Jovanovic (1982) shows that information does not fully unravel in equilibrium. In this case, the seller would rather stay silent for qualities that are below a certain threshold. However, disclosure is socially wasteful because it is purely redistributive. In other words, the seller engages in excessive information disclosure when it is costly to disclose quality.

The "classical literature" assumes that consumers have unit demands with identical reservation prices.[2] Under this assumption, the total quantity consumed in equilibrium is the same for all disclosure policies. Therefore, for any positive disclosure cost (regardless of how small it is), all policies other than non-disclosure involve socially excessive levels of information disclosure.[3] However, it is not clear if this same result carries over to situations in which the market demand is downward-sloping. In such a case, disclosure generally leads to a change in the equilibrium quantity consumed and is therefore not purely redistributive anymore. Since the seller ignores any consumer surplus effects when making his disclosure decision, disclosure will be socially inefficient. However,

[1] To see the intuition, suppose there are $n > 1$ possible quality levels. If the product has the highest possible quality, the seller would surely reveal it because staying silent would simply induce a lower perceived quality and a lower willingness-to-pay. Thus, when consumers see non-disclosure, they will place zero probability on the highest quality and as a result the expected quality in case of non-disclosure will be strictly below the second highest quality. The seller, therefore, would as well choose full disclosure when the product has the second highest quality. Continuing in a similar fashion, one can eliminate all qualities but the lowest one. As a result, if quality information is ever withheld, it must be that the product has the lowest possible quality.

[2] By "classical literature," I refer to Grossman (1981), Grossman and Hart (1980), Milgrom (1981), and Jovanovic (1982).

[3] Note that when disclosure is costless, all disclosure policies are welfare-equivalent.

it is *a priori* unclear if this inefficiency is in the form of excessive or insufficient disclosure.

As noted earlier, when the cost of disclosure is positive, there will be an interior threshold quality level that leaves the seller indifferent between disclosing and not. If full disclosure of this particular threshold quality strictly raises the underlying consumer surplus, however, a social planner can improve the *ex-ante* expected aggregate welfare by marginally lowering this threshold.[4] In such a situation, the level of voluntary information disclosure will be socially insufficient.

To address this question, I consider a simple model with a single seller and a single consumer who has a linear downward-sloping demand. I show that the excessive disclosure result of the "classical literature" is still valid when all quality levels are socially desirable and the consumer's prior beliefs are uniform over the possible values of quality. A social planner cannot improve expected welfare by mandating that the seller disclose a larger set of qualities. In other words, the consumer surplus effect mentioned above is not large enough to make disclosure insufficient.

Next, I identify two situations under which voluntary disclosure may be socially insufficient; both of these situations arise when the demand curve is sufficiently elastic: The first one is when there are quality levels that are too low to generate any positive demand by the consumer. In this case, non-disclosure may lead the consumer to reduce her demand to zero even if the actual quality normally induces a positive demand when disclosed (or, in other words, even if the actual quality is socially desirable). The second one is when the consumer's prior beliefs place sufficiently higher weight on lower qualities. Now, a failure to disclose quality leads to a severely bad perception of the product, thereby significantly lowering the consumer's demand. As a result, the amount of forgone consumer surplus is sufficiently large.

The remainder of the paper is organized as follows: In the next section, I briefly review the literature. Section 3 introduces the main setup. In section 4, I analyze the equilibrium and the socially efficient levels of disclosure for three different scenarios. Finally, in section 5, I conclude. All proofs are relegated to the Appendix.

[4] Given that the seller charges a higher price under full disclosure, consumer surplus would increase only if the quantity consumed goes up sufficiently.

2 Related literature

There is a large literature that analyzes verifiable information disclosure. As mentioned in the introduction, the classical articles are Grossman (1981), Grossman and Hart (1980), Milgrom (1981), and Jovanovic (1982). Important extensions include Dye (1985), who introduces the possibility that the seller does not know the quality either, Matthews and Postlewaite (1985), who endogenize the decision to acquire information about quality, Fishman and Hagerty (1990), who analyze the level of discretion a seller should be allowed in choosing how much information to disclose, Shin (1994), who incorporates uncertainty about the degree of information the seller possesses, and Board (2009), Cheong and Kim (2004), and Milgrom and Roberts (1986), who analyze quality disclosure in competitive environments.[5]

In a recent paper, Daughety and Reinganum (2008) argue that the alternative to disclosure should not be viewed as non-disclosure. They introduce price signaling as an alternative means of revealing quality in a framework where demand is downward-sloping, disclosure is costly, and marginal cost of production is strictly increasing in quality. They characterize an equilibrium in which the seller chooses to reveal qualities below a threshold via price signaling and those above via direct disclosure. They also find that the seller uses direct disclosure insufficiently from a social point of view for any positive disclosure cost.

In order to understand how much of this insufficient disclosure result is driven by the assumption of downward-sloping demand, Daughety and Reinganum (2008) construct an example in which the marginal cost of production is independent of quality, thus making price signaling infeasible. In this case, they find that information disclosure is socially insufficient when the cost of disclosure is above a threshold.[6] Based on this finding, they conclude that the general finding by the classical literature that information disclosure is excessive is importantly driven by that literature's assumption that consumers have unit demands with identical reservation prices and that such a sweeping result is unlikely to be found in models with downward-sloping demand.

In the benchmark scenario that I consider in section 4.1 – which has the same setup as the example Daughety and Reinganum (2008) construct – I show that there is excessive information disclosure for all parameter values. This finding demonstrates that the second part of Daughety and

[5] See Dranove and Jin (2010) for an excellent survey on verifiable information disclosure.

[6] See Proposition 4 (p. 985) in Daughety and Reinganum (2008).

Reinganum's conclusion is incorrect and shows that the excessive disclosure finding of the classical literature can extend to more general models with downward-sloping demand.[7] However, when the demand curve is sufficiently elastic, there may be insufficient disclosure as I show in sections 4.2 and 4.3. In this regard, the insight of Daughety and Reinganum (2008) remains true.

3 Main setup

A profit-maximizing monopoly seller (he) offers a product for sale, which is characterized by its quality: $\theta \in [0, 1]$. The quality θ is exogenously given and is privately known by the seller. To make the results comparable to the existing literature on verifiable information disclosure, I assume that the marginal cost of production does not depend on quality and is therefore set to zero without any loss of generality.[8] On the other side of the market, there is a single consumer (she) who is unable to observe θ prior to purchase. I consider a prior belief structure that potentially places higher weight on lower qualities. To make the analysis simple and tractable, I assume that the consumer places a probability $\mu \geq 0$ on the event that $\theta = 0$, and a probability $(1 - \mu)$ on the event that θ is uniformly distributed over $(0, 1]$. This structure allows me to alter the skewness of beliefs by changing one parameter only.

The consumer's utility is quadratic in the quantity of the product consumed and quasilinear in all other goods. In particular, for a given perceived quality $\tilde{\theta}$ and a price p, the consumer's utility is given by

$$U(p, q, \tilde{\theta}) = (\alpha - (1 - \tilde{\theta})\delta)q - \frac{\beta}{2}q^2 + I - pq. \tag{1}$$

Here, I is the consumer's income and $(\alpha - (1 - \tilde{\theta})\delta)q - \frac{\beta}{2}q^2$ is her utility from consuming q units of the product (thus, $I - pq$ is the amount spent on all other goods). The marginal utility of consuming the very first unit of the product is $\alpha - (1 - \tilde{\theta})\delta$, where a higher δ implies a more quality-sensitive consumer. If, for instance, δ is high and the perceived quality is low, then even the first unit of consumption may bring a negative utility. The term β, on the other hand, measures

[7] See the following website for their subsequent recognition that their claim is too sweeping: http://www.vanderbilt.edu/econ/faculty/Daughety/Daughetypreviouspubs.html.

[8] If, for instance, marginal cost increases with quality, then price may carry a signaling role in situations where quality is not revealed through direct disclosure. Daughety and Reinganum (2008) focus on fully separating prices in this context and characterize an equilibrium in which both price signaling and direct disclosure are used. There is, however, a plethora of other equilibria that involve semi-separating or pooling pricing strategies in the non-disclosure subgame. When marginal cost does not depend on quality, these problems do not arise; all seller types that choose non-disclosure charge the same price.

the constant rate at which marginal utility diminishes. I assume that α, δ, $\beta > 0$ and that I is large enough so as to ensure $I - pq > 0$.

Before a transaction takes place, the seller may choose fully to disclose θ through a credible means at a cost $D > 0$. In this case, $\tilde{\theta} = \theta$. If the seller chooses to stay silent, on the other hand, the consumer makes rational inferences about θ. The consumer knows all aspects of the model (including the cost of disclosure D) and thus can rationally infer the subset of θ for which the seller stays silent.[9] As a tie-breaking rule, I assume that the seller stays silent when he is indifferent between disclosing and not.

For a given $\tilde{\theta}$ and a price p, the consumer maximizes her utility by choosing the optimal quantity $q(p, \tilde{\theta})$. Taking the derivative of $U(p, q, \tilde{\theta})$ with respect to q and setting it equal to zero yields the following linear demand function:

$$q(p, \tilde{\theta}) = \begin{cases} 0 & , \tilde{\theta} \leq 1 - \frac{\alpha}{\delta} \\ \frac{\alpha - (1-\tilde{\theta})\delta - p}{\beta} & , \tilde{\theta} > 1 - \frac{\alpha}{\delta} \end{cases} . \tag{2}$$

Thus, the price intercept of the demand curve is $\max\left\{0, \alpha - (1 - \tilde{\theta})\delta\right\}$, which depends on α, δ and the perceived quality $\tilde{\theta}$. If the product is of the highest possible quality, then the price intercept is simply α. As the perceived quality goes down, the intercept shifts down at a rate δ. However, if it is too low – i.e., if $\tilde{\theta} \leq 1 - \frac{\alpha}{\delta}$ – then the consumer has no demand for the product at any positive price. In this case, low qualities of the product are socially undesirable. This is certainly relevant only when $\alpha \leq \delta$, which implies $1 - \frac{\alpha}{\delta} \geq 0$. Otherwise, if $\alpha > \delta$, the product is socially desirable. Also note that the price elasticity of demand is $\frac{-p}{\alpha - \left(1-\tilde{\theta}\right)\delta - p}$, so a lower α or $\tilde{\theta}$, or a higher δ are all associated with a more elastic demand.

4 Equilibrium vs. socially optimal levels of disclosure

In this section, I consider three scenarios with different parameter restrictions: The first one is a benchmark scenario in which all qualities are socially desirable (i.e., $\alpha > \delta$) and the consumer's prior beliefs are uniform over $[0, 1]$ with no mass point at the lowest quality (i.e., $\mu = 0$). In each one of the other two scenarios, I relax one of these two assumptions while keeping the other one the same. In the second scenario, the prior beliefs are uniform over $[0, 1]$, but low qualities of the

[9] One can interpret this as the consumer's learning through a process of trial and error in observing when the seller discloses and when he does not.

product are undesirable. In the third scenario, prior beliefs place a strictly positive probability on $\theta = 0$, while all qualities are socially desirable.

Before proceeding with the benchmark scenario, it is helpful to determine and discuss the general conditions for the equilibrium and the socially optimal levels of disclosure. Given the demand function in (2), a type-θ seller maximizes (gross of disclosure cost)

$$\pi(p, \theta, \tilde{\theta}) = \frac{p(\alpha - (1 - \tilde{\theta})\delta - p)}{\beta}. \tag{3}$$

For a given perceived quality $\tilde{\theta} > 1 - \frac{\alpha}{\delta}$, maximizing $\pi(p, \theta, \tilde{\theta})$ with respect to p yields

$$p(\tilde{\theta}) = \frac{\alpha - (1 - \tilde{\theta})\delta}{2}, \tag{4}$$

$$q(\tilde{\theta}) = \frac{\alpha - (1 - \tilde{\theta})\delta}{2\beta}, \tag{5}$$

$$\pi(\tilde{\theta}) = \frac{(\alpha - (1 - \tilde{\theta})\delta)^2}{4\beta}. \tag{6}$$

If the perceived quality is below or equal to $1 - \frac{\alpha}{\delta}$, the seller does not make any production and earns zero profits.

It is important to note from equation (5) that $q(\tilde{\theta})$ is strictly increasing in $\tilde{\theta}$ (recall that it was assumed $\delta > 0$). This means that, for a given quality θ, disclosure strictly raises the equilibrium quantity that is purchased by the consumer, and in turn leads to a strictly higher level of aggregate welfare (gross of the disclosure cost). This is precisely how an analysis with a downward-sloping demand differs from the classical approach in terms of the social efficiency of disclosure; in the latter, disclosure does not change the equilibrium level of consumption and thus is purely redistributive.

When disclosure involves a positive cost, there will be a marginal quality level θ^* that leaves the seller indifferent between disclosing and not. This is given by

$$\pi\left(\theta^*\right) - D = \pi\left(\mathbb{E}\left[\theta \mid \theta \leq \theta^*\right]\right), \tag{7}$$

where the left-hand side is the profit the seller earns in case of disclosure and the right-hand side is the profit when he stays silent and the consumer infers that $\theta \leq \theta^*$. If there is an interior θ^* that solves (7), then the seller discloses all qualities above θ^* while concealing those under θ^*. If it is the case that $\pi\left(1\right) - D < \pi\left(\mathbb{E}\left[\theta\right]\right)$, then the seller conceals all quality realizations, so $\theta^* = 1$. This happens when the disclosure cost is sufficiently high. When $D = 0$, on the other hand, it is clear from (7) that all qualities that generate positive profits are disclosed. Hence, in this case, $\theta^* = \max\left\{0, 1 - \frac{\alpha}{\delta}\right\}$.

The socially optimal level of information disclosure can similarly be summarized by a threshold quality level θ^s such that all qualities above this threshold are disclosed while the ones below are concealed. When the true quality is θ and the perceived quality is $\tilde{\theta}$, the aggregate welfare gross of disclosure cost is $U(p(\tilde{\theta}), q(\tilde{\theta}), \theta) + \pi(\tilde{\theta})$. Using the utility function given in equation (1), observing that price is simply a transfer between the seller and the consumer, and noting that there is a discrete probability μ that $\theta = 0$, one can express the *ex-ante* expected aggregate welfare $W(\theta^s)$ for a given θ^s as

$$\begin{aligned} W(\theta^s) \quad = \quad & \mu\left[(\alpha - \delta)\, q(\tilde{\theta}^s) - \frac{\beta}{2}\left(q(\tilde{\theta}^s)\right)^2 + I\right] \\ & + (1-\mu)\int_0^{\theta^s}\left[(\alpha - (1-\theta)\,\delta)\, q(\tilde{\theta}^s) - \frac{\beta}{2}\left(q(\tilde{\theta}^s)\right)^2 + I\right] d\theta \\ & + (1-\mu)\int_{\theta^s}^{1}\left[(\alpha - (1-\theta)\,\delta)\, q(\theta) - \frac{\beta}{2}\left(q(\theta)\right)^2 + I - D\right] d\theta, \end{aligned} \tag{8}$$

where $\tilde{\theta}^s = \mathbb{E}\left[\theta \mid \theta \leq \theta^s\right]$. One can then find the optimal value of θ^s by maximizing $W(\theta^s)$ with respect to θ^s. Once again, when the cost of disclosure is sufficiently high, this maximization problem will imply $\theta^s = 1$. Similarly, when disclosure is costless, it will imply $\theta^s = \max\left\{0, 1 - \frac{\alpha}{\delta}\right\}$. This latter observation follows from the fact that $q(\tilde{\theta})$ is strictly increasing in $\tilde{\theta}$ and a higher quantity implies a strictly higher welfare. Hence, the equilibrium and the socially optimal quality levels coincide when disclosure is costless or when it is too costly.

As discussed earlier, the classical literature argues that when $D > 0$, no disclosure should take place from a social point of view (i.e., $\theta^s = 1$). This is due to the fact that disclosure does not change the quantity consumed in equilibrium and hence is purely redistributive. However, when the demand curve is downward-sloping, there will be an interior θ^s (unless D is too large), and it is hard to say *a priori* if this is above or below θ^*.

4.1 Benchmark scenario: $\mu = 0$ and $\alpha > \delta$

I start off with a benchmark scenario in which the consumer's prior beliefs are uniformly distributed over $[0, 1]$ and all qualities are socially desirable. This setup also corresponds to the example in

Daughety and Reinganum [2008]. Given the uniform priors, $\mathbb{E}[\theta \mid \theta < \theta^*] = \frac{\theta^*}{2}$. Using the profit function given in equation (6), the indifference condition given in (7) becomes

$$\frac{(\alpha - (1-\theta^*)\delta)^2}{4\beta} - D = \frac{\left(\alpha - \left(1 - \frac{\theta^*}{2}\right)\delta\right)^2}{4\beta}, \tag{9}$$

which, after simplifying, reduces to

$$(\theta^*)^2 + \frac{4(\alpha-\delta)}{3\delta}\theta^* - \frac{16\beta D}{3\delta^2} = 0. \tag{10}$$

Solving this expression for θ^* yields

$$\theta^* = \frac{1}{2}\left(-\frac{4(\alpha-\delta)}{3\delta} + \sqrt{\left(\frac{4(\alpha-\delta)}{3\delta}\right)^2 + \frac{64\beta D}{3\delta^2}}\right). \tag{11}$$

Reorganizing and noting that θ^* cannot exceed 1 leads to

$$\theta^* = \begin{cases} \frac{2(\alpha-\delta)}{3\delta}\left[\sqrt{1+\frac{12\beta D}{(\alpha-\delta)^2}} - 1\right] & \text{, if } D < \frac{(4\alpha-\delta)\delta}{16\beta} \\ 1 & \text{, if } D \geq \frac{(4\alpha-\delta)\delta}{16\beta} \end{cases}. \tag{12}$$

As described earlier, the socially optimal threshold quality level θ^s is found by maximizing the *ex-ante* expected welfare function $W(\theta^s)$ given in equation (8). In section A.1 of the Appendix, I show for a general $\mu \in [0,1]$ that this maximization problem leads to the following condition:

$$\frac{3\delta^2}{8\beta}\left(\theta^s - \mathbb{E}[\theta \mid \theta \leq \theta^s]\right)^2 = D. \tag{13}$$

For the current benchmark scenario, $\mathbb{E}[\theta \mid \theta \leq \theta^s] = \frac{\theta^s}{2}$, so condition (13) implies[10]

$$\theta^s = \begin{cases} \sqrt{\frac{32\beta D}{3\delta^2}} & \text{, if } D < \frac{3\delta^2}{32\beta} \\ 1 & \text{, if } D \geq \frac{3\delta^2}{32\beta} \end{cases}. \tag{14}$$

A simple comparison of θ^* and θ^s in expressions (12) and (14) yields that $\theta^* = \theta^s = 1$ when $D \geq \frac{(4\alpha-\delta)\delta}{16\beta}$ and that $\theta^* < \theta^s = 1$ when $\frac{3\delta^2}{32\beta} \leq D < \frac{(4\alpha-\delta)\delta}{16\beta}$. Again, it is straightforward to show that $\theta^* < \theta^s$ when $0 < D < \frac{3\delta^2}{32\beta}$, which leads to the following result:

Proposition 1. *When the consumer's prior beliefs for quality are uniformly distributed over* $[0,1]$ *and* $\alpha > \delta$ *so that all qualities are socially desirable, the seller never engages in insufficient disclosure.*

[10] Alternatively, one can simply evaluate $W(\theta^s)$ at $\mu = 0$ and $\tilde{\theta}^s = \frac{\theta^s}{2}$, and then maximize the resulting expression with respect to θ^s. I adopt the current approach because I use it later in scenario 3 as well.

As mentioned earlier, for a given quality θ, disclosure strictly raises the equilibrium quantity purchased and in turn the area under the demand curve. However, after accounting for the cost of disclosure, the aggregate welfare may actually go down. While the equilibrium threshold quality that leaves the seller indifferent between disclosing and not offsets the profit gain and the cost of disclosure, if the consumer surplus under disclosure is lower for this threshold quality level, then disclosure will overall lead to a lower level of welfare.

The consumer surplus may indeed decrease under disclosure. As a thought experiment, consider an extreme example and take the demand curve given by $q(p, \tilde{\theta}) = 1 - \frac{p}{\tilde{\theta}}$. Under this demand curve, the quantity consumed in equilibrium is always equal to $\frac{1}{2}$ regardless of the perceived quality level $\tilde{\theta}$. For a given actual quality θ, the realized gross surplus (net of disclosure cost) is the area under the *true* demand curve $q(p, \theta)$ and to the left of the consumed quantity (again, regardless of the perceived quality level $\tilde{\theta}$). Since the equilibrium quantity consumed does not depend on the particular disclosure policy, disclosure is socially wasteful for all quality levels. However, if there is an interior equilibrium threshold quality level θ^*, the seller charges a higher price when disclosing θ^* than concealing it, and thus the net consumer surplus is lower.

For the particular demand function I consider in this paper, disclosure always expands the demand curve and leads to a strictly higher equilibrium quantity purchased. However, if the increase in the quantity purchased is not sufficiently large, the resulting change in the net consumer surplus may be negative. Suppose the actual quality is θ and the perceived quality is $\tilde{\theta}$. Using the utility function given in equation (1), the equilibrium quantity given in equation (5) and equilibrium price given in equation (4), one can easily calculate the change in consumer surplus due to disclosure:

$$U(p(\theta), q(\theta), \theta) - U(p(\tilde{\theta}), q(\tilde{\theta}), \theta) = \frac{\delta(\theta - \tilde{\theta})}{4\beta} \left[-\left(\alpha - \left(1 - \frac{3\tilde{\theta} - \theta}{2} \right) \delta \right) \right]. \tag{15}$$

Note that this expression measures the difference in the *realized* net utility (and hence the realized consumer surplus) for a given quality level θ. The first term $U(p(\theta), q(\theta), \theta)$ measures the net utility when θ is disclosed and thus the actual and the perceived qualities overlap. The second term $U(p(\tilde{\theta}), q(\tilde{\theta}), \theta)$, on the other hand, measures the net utility when the actual quality is θ but the perceived quality is $\tilde{\theta}$ because the seller has not disclosed θ. The firm charges $p(\tilde{\theta})$ knowing that the consumer will base her purchase decision on perceived quality $\tilde{\theta}$, and the consumer purchases an amount $q(p(\tilde{\theta}), \tilde{\theta})$. After purchase, however, the consumer realizes that the true quality is θ.

Thus, the relevant net utility in this case is $U(p(\tilde{\theta}), q(\tilde{\theta}), \theta)$.

Given that the prior beliefs are uniform over $[0,1]$, for any equilibrium threshold quality level $\theta^* > 0$, the expression in (15) equals

$$U(p(\theta^*), q(\theta^*), \theta^*) - U(p(\theta^*/2), q(\theta^*/2), \theta^*) = -\frac{\delta\theta^*}{8\beta}\left(\alpha - \left(1 - \frac{\theta^*}{4}\right)\delta\right), \tag{16}$$

which is strictly negative for any θ^*. Hence, from a social planner's point of view, marginally lowering θ^* in fact lowers consumer surplus (and thus welfare). This means that the seller engages in excessive disclosure. It should be noted that a lower quality threshold necessarily means that the consumer's perception of quality will be lower in case of non-disclosure. This in turn will lead to a welfare loss for those qualities that are concealed, which makes the excessive disclosure result even more pronounced for the current benchmark scenario.

Proposition 1 has an important implication; *The unit demand assumption is not critical for the excessive disclosure result.* The same result obtains in more general environments with a linear downward-sloping demand. However, the uniformity of the prior beliefs as well as the fact that even the lowest quality generates a positive demand are important factors behind this result. Both of these factors ensure that non-disclosure of quality does not lower the quantity consumed too much, thereby keeping the demand curve relatively inelastic.[11] As I show in the next two scenarios, this observation is quite important for the results; if the demand curve becomes sufficiently elastic under non-disclosure, then it is possible that the seller engages in excessive information disclosure.

4.2 Scenario 2: $\mu = 0$ and $\alpha \leq \delta$

In this scenario, the prior density function is still uniform over $[0,1]$, but now $\alpha \leq \delta$, so low types of the seller cannot generate positive profits. It also means that the demand curve is relatively more elastic and as a result non-disclosure lowers the quantity consumed relatively more. One can imagine a situation in which all possible quality levels are equally probable from the consumer's point of view, but sufficiently low qualities are undesirable and therefore generate zero demand. This may, for instance, be due to higher potential hazards that lower quality products are associated with. For the remainder of this subsection, I assume $2\alpha > \delta$, which ensures a positive demand in case the seller adopts a no-disclosure strategy (i.e., when the perceived quality is $\frac{1}{2}$).

[11] Recall that a lower α or $\tilde{\theta}$, or a higher δ are all associated with a more elastic demand.

When the perceived quality $\mathbb{E}[\theta \mid \theta \leq \theta^*]$ is above $1-\frac{\alpha}{\delta}$, the calculation of the equilibrium threshold quality follows the same steps as in the benchmark scenario.[12] However, for low values of D, the value of θ^* in equation (12) may fall below $2\left(1-\frac{\alpha}{\delta}\right)$: the threshold quality level, which implies a perceived quality of $1-\frac{\alpha}{\delta}$ and thus $q=0$ in case of non-disclosure. This happens when $D<\frac{(\delta-\alpha)^2}{4\beta}$, so we need to recalculate θ^* for these values of D. If we account for zero profits in case of non-disclosure, the indifference condition in (7) becomes

$$\frac{\left(\alpha-\left(1-\theta^*\right)\delta\right)^2}{4\beta}-D=0, \tag{17}$$

which implies $\theta^*=1-\frac{\alpha-2\sqrt{\beta D}}{\delta}$ in this region. Hence, the equilibrium value of θ^* in this scenario is given by

$$\theta^*=\begin{cases} 1-\frac{\alpha-2\sqrt{\beta D}}{\delta} & , D<\frac{(\delta-\alpha)^2}{4\beta} \\ \frac{2(\delta-\alpha)}{3\delta}\left[\sqrt{1+\frac{12\beta D}{(\delta-\alpha)^2}}+1\right] & , \frac{(\delta-\alpha)^2}{4\beta}\leq D<\frac{(4\alpha-\delta)\delta}{16\beta} \\ 1 & , D\geq\frac{(4\alpha-\delta)\delta}{16\beta} \end{cases}. \tag{18}$$

For the socially optimal threshold quality level, we follow similar steps as above. In particular, for $D<\frac{3(\delta-\alpha)^2}{8\beta}$, the value of θ^s in the benchmark scenario, given in equation (14), leads to a perceived quality that is less than $1-\frac{\alpha}{\delta}$, which implies $q(\tilde{\theta})=0$. This means that the welfare function in this region is

$$W\left(\theta^s\right)=\int_{\theta^s}^{1}\left[\left(\alpha-(1-\theta)\delta\right)q(\theta)-\frac{\beta}{2}\left(q(\theta)\right)^2-D\right]d\theta. \tag{19}$$

One can easily find that the value of θ^s that maximizes this expression is $\theta^s=1-\frac{\alpha-2\sqrt{2\beta D/3}}{\delta}$. Hence,

$$\theta^s=\begin{cases} 1-\frac{\alpha-2\sqrt{2\beta D/3}}{\delta} & , D<\frac{3(\delta-\alpha)^2}{8\beta} \\ \sqrt{\frac{32\beta D}{3\delta^2}} & , \frac{3(\delta-\alpha)^2}{8\beta}\leq D<\frac{3\delta^2}{32\beta} \\ 1 & , D\geq\frac{3\delta^2}{32\beta} \end{cases}. \tag{20}$$

Again, a simple comparison of θ^* and θ^s given in (18) and (20) yields the following result:

Proposition 2. *When the consumer's prior beliefs for quality are uniformly distributed over* $[0,1]$, *but* $\frac{\delta}{2}<\alpha\leq\delta$ *so that low qualities are socially undesirable, the seller engages in insufficient disclosure (i) for all* $D\in\left(0,\frac{(4\alpha-\delta)\delta}{16\beta}\right)$ *if* $\alpha\leq\frac{5\delta}{8}$, *or (ii) for all* $D\in\left(0,\frac{2(\delta-\alpha)^2}{3\beta}\right)$ *if* $\frac{5\delta}{8}<\alpha\leq\delta$.[13]

[12] The only difference is that the term $\left(\frac{4(\alpha-\delta)}{3\delta}\right)^2$ in equation (11) will come out of the square root as $\frac{4(\delta-\alpha)}{3\delta}$.

[13] See the proof for further details.

As explained in the beginning of this subsection, when α is smaller than δ, the demand curve is relatively more elastic and therefore the reduction in quantity consumed due to non-disclosure is relatively higher. The insufficient disclosure result is easiest to see when $D < \frac{(\delta-\alpha)^2}{4\beta}$. In this case, even when the true quality level is socially desirable, the seller may choose non-disclosure (i.e., when $1 - \frac{\alpha}{\delta} < \theta < 1 - \frac{\alpha - 2\sqrt{\beta D}}{\delta}$), which leads to a perceived quality below $1 - \frac{\alpha}{\delta}$ and thus to zero quantity consumed. From a social point of view, however, a socially desirable quality level generates a strictly positive consumer surplus if disclosed. Therefore, marginally lowering θ^* enhances welfare when $D < \frac{(\delta-\alpha)^2}{4\beta}$. This holds true regardless of the value of $\frac{\alpha}{\delta}$.

In the limit when $\alpha \to \frac{\delta}{2}$, the second row in expression (18) disappears, so non-disclosure always induces a perceived quality level that is below $1 - \frac{\alpha}{\delta}$. Therefore, in this case, disclosure is insufficient for all disclosure costs.[14] Similarly, when $\alpha = \delta$, the second row in equation (18) implies $\theta^* = \sqrt{\frac{16\beta D}{3\delta^2}}$ for $D < \frac{3\delta^2}{16\beta}$, implying excessive disclosure compared with θ^s. Hence, when α is not too much lower than δ, the insufficient disclosure result obtains only for low enough disclosure costs.

4.3 Scenario 3: $\mu > 0$ and $\alpha > \delta$

In this scenario, the product is socially valuable (i.e., $\alpha > \delta$), but now there is a discrete probability $\mu > 0$ that $\theta = 0$. This leads to a density function that is skewed to the right. The skewness can be varied by changing the value of μ. An example is when the consumer has tried several other products with the seller and has had bad experience with some (or most) of them.

When $\mu > 0$, $\mathbb{E}[\theta \mid \theta < \theta^*]$ is given by

$$\mathbb{E}[\theta \mid \theta < \theta^*] = \left(\frac{(1-\mu)\theta^*}{\mu + (1-\mu)\theta^*} \right) \frac{\theta^*}{2}. \tag{21}$$

Unfortunately, this expression turns the indifference condition given in (7) into a 4^{th}-order polynomial, which makes difficult the reaching of a closed-form solution for θ^*. Therefore, I solve for the equilibrium and the socially optimal threshold quality levels for the limiting case $\mu \to 1$, and then interpret the findings for $\mu \in (0,1)$. Under this assumption, staying silent leads to a perceived quality that is approximately zero for any threshold quality level. Rewriting the indifference condition (7) for this limiting case yields

$$\frac{(\alpha - (1-\theta^*)\,\delta)^2}{4\beta} - D = \frac{(\alpha - \delta)^2}{4\beta}, \tag{22}$$

[14] This observation also implies that disclosure is always insufficient when $\alpha \leq \frac{\delta}{2}$ unless D is zero or sufficiently large.

which, after solving for θ^*, leads to

$$\theta^* = \begin{cases} \frac{(\alpha-\delta)}{\delta}\left[\sqrt{1+\frac{4\beta D}{(\alpha-\delta)^2}}-1\right] & \text{, if } D < \frac{\alpha^2-(\alpha-\delta)^2}{4\beta} \\ 1 & \text{, if } D \geq \frac{\alpha^2-(\alpha-\delta)^2}{4\beta} \end{cases} . \tag{23}$$

To find the socially optimal threshold quality level, we simply use condition (13), which was also used in the benchmark scenario: $\frac{3\delta^2}{8\beta}\left(\theta^s - \mathbb{E}\left[\theta \mid \theta \leq \theta^s\right]\right)^2 = D$.[15] Note that the only term in this expression that depends on μ is $\mathbb{E}\left[\theta \mid \theta \leq \theta^s\right]$, which goes to 0 as μ goes to 1. Hence,

$$\theta^s = \begin{cases} \sqrt{\frac{8\beta D}{3\delta^2}} & \text{, if } D < \frac{3\delta^2}{8\beta} \\ 1 & \text{, if } D \geq \frac{3\delta^2}{8\beta} \end{cases} . \tag{24}$$

Comparing this with θ^* given in expression (23) yields the following result:

Proposition 3. *When $\mu \to 1$ and $\alpha > \delta$ so that all qualities are socially desirable, the seller engages in insufficient disclosure if $\frac{\alpha}{\delta} < \frac{5}{4}$ and $\frac{6(\alpha-\delta)^2}{\beta} < D < \frac{3\delta^2}{8\beta}$.*

To see the intuition for this result, suppose first that $\alpha = \delta$. In this case, the demand curve is relatively more elastic and, given that $\mu \to 1$, non-disclosure of any quality leads to a quantity that is equal to zero (and thus zero profits for the seller). Since the seller ignores the consumer surplus when deciding whether to disclose or not, marginally lowering θ^* enhances the *ex-ante* welfare for any $D \in \left(0, \frac{3\delta^2}{8\beta}\right)$. When α is sufficiently higher than δ, on the other hand, the demand curve becomes more inelastic and as a result the level of information disclosed by the seller becomes excessive.

Using equation (15) and noting that non-disclosure leads to a perceived quality of zero, disclosure of a particular θ improves the consumer surplus by an amount

$$U(p(\theta), q(\theta), \theta) - U(p(0), q(0), \theta) = -\frac{\delta\theta}{4\beta}\left(\alpha - \left(1+\frac{\theta}{2}\right)\delta\right).$$

The term in the parenthesis is surely positive when $\frac{\alpha}{\delta} > \frac{3}{2}$, implying that disclosure is always excessive in this range. In the alternative case when $1 < \frac{\alpha}{\delta} \leq \frac{3}{2}$, marginally lowering high values of θ^* may be beneficial. However, as discussed earlier, this will have two opposing effects. On the one hand, the consumer will strictly benefit if the actual quality level happens to be in the region where it was not disclosed before but is disclosed now. On the other hand, when the actual quality

[15] For a formal derivation of this condition, see section A.1 of the Appendix.

is in the non-disclosure range, the consumer's perception of quality will go down, which reduces welfare. Overall these two effects work against each other and the proposition establishes that the former effect dominates when $\frac{\alpha}{\delta} < \frac{5}{4}$, thus leading to the insufficient disclosure result.

The above analysis ensures that there are values of α, δ and D for which market disclosure is insufficient even when μ is strictly smaller than 1. However, based on the results obtained in the benchmark scenario, the value of μ needs to be sufficiently high. Hence, uniformity of the prior density function is an important factor behind the excessive disclosure result of the benchmark scenario.

5 Conclusion

In this paper, I have analyzed whether the excessive disclosure result of the "classical literature" extends to a more general setting with a linear downward-sloping demand function. I find that it does extend unless the demand curve is highly elastic. If the demand curve is sufficiently elastic, then there are at least two scenarios under which disclosure may be *insufficient* from a social perspective: when there are quality levels that generate zero demand, and when the prior beliefs place higher probabilities on lower qualities. An extension of this analysis might focus on the necessary conditions for insufficient disclosure in an environment with a more general demand function.

A Appendix

A.1 Maximization of $W(\theta^s)$ for $\alpha > \delta$ and $\mu \in [0,1]$

In this section of the Appendix, I derive and simplify the first-order condition that is associated with the maximization of the *ex-ante* expected welfare function given in equation (8) for $\alpha > \delta$ and any $\mu \in [0,1]$. For convenience, rewrite equation (8):

$$\begin{aligned} W(\theta^s) &= \mu\left[(\alpha-\delta)\, q(\tilde{\theta}^s) - \frac{\beta}{2}\left(q(\tilde{\theta}^s)\right)^2\right] \\ &+ (1-\mu)\int_0^{\theta^s}\left[(\alpha-(1-\theta)\,\delta)\, q(\tilde{\theta}^s) - \frac{\beta}{2}\left(q(\tilde{\theta}^s)\right)^2\right] d\theta \\ &+ (1-\mu)\int_{\theta^s}^{1}\left[(\alpha-(1-\theta)\,\delta)\, q(\theta) - \frac{\beta}{2}\left(q(\theta)\right)^2 - D\right] d\theta. \end{aligned}$$

The second term above can be rewritten as

$$(1-\mu)\,\theta^s\left[\left(\alpha-\left(1-\frac{\theta^s}{2}\right)\delta\right)q(\tilde{\theta}^s)-\frac{\beta}{2}\left(q(\tilde{\theta}^s)\right)^2\right].$$

Adding this term to the first term and noting that $\tilde{\theta}^s=\left(\frac{(1-\mu)\theta^s}{\mu+(1-\mu)\theta^s}\right)\frac{\theta^s}{2}$ leads to

$$(\mu+(1-\mu)\,\theta^s)\left[\left(\alpha-(1-\tilde{\theta}^s)\delta\right)q(\tilde{\theta}^s)-\frac{\beta}{2}\left(q(\tilde{\theta}^s)\right)^2\right].$$

Given that $q(\tilde{\theta}^s)=\frac{\alpha-\left(1-\tilde{\theta}^s\right)\delta}{2\beta}$, this term can further be simplified as

$$\frac{3}{8\beta}\left(\mu+(1-\mu)\,\theta^s\right)\left(\alpha-(1-\tilde{\theta}^s)\delta\right)^2.$$

Similarly, given that $q\left(\theta\right)=\frac{\alpha-(1-\theta)\delta}{2\beta}$, the third term in (8) can be rewritten as

$$(1-\mu)\int_{\theta^s}^{1}\left[\frac{3}{8\beta}\left(\alpha-\left(1-\theta\right)\delta\right)^2-D\right]d\theta.$$

Hence,

$$W\left(\theta^s\right)=\frac{3}{8\beta}\left(\mu+(1-\mu)\,\theta^s\right)\left(\alpha-(1-\tilde{\theta}^s)\delta\right)^2+(1-\mu)\int_{\theta^s}^{1}\left[\frac{3}{8\beta}\left(\alpha-\left(1-\theta\right)\delta\right)^2-D\right]d\theta.$$

Maximizing this expression with respect to θ^s yields the following first-order condition:

$$\begin{aligned}&\frac{3}{8\beta}\left(1-\mu\right)\left(\alpha-(1-\tilde{\theta}^s)\delta\right)^2+\frac{6\delta}{8\beta}\left(\mu+(1-\mu)\,\theta^s\right)\left(\alpha-(1-\tilde{\theta}^s)\delta\right)\frac{d\tilde{\theta}^s}{d\theta^s}\\ =\;&\left(1-\mu\right)\left[\frac{3}{8\beta}\left(\alpha-\left(1-\theta^s\right)\delta\right)^2-D\right]\end{aligned}$$

Note that

$$\begin{aligned}\frac{d\tilde{\theta}^s}{d\theta^s} &= \frac{(1-\mu)\,\theta^s}{2\left(\mu+(1-\mu)\,\theta^s\right)}+\frac{\mu\left(1-\mu\right)\theta^s}{2\left(\mu+(1-\mu)\,\theta^s\right)^2}\\ &= \frac{(1-\mu)}{\mu+(1-\mu)\,\theta^s}\left[\frac{\theta^s}{2}+\frac{\mu\theta^s}{2\left(\mu+(1-\mu)\,\theta^s\right)}\right]\\ &= \frac{(1-\mu)}{\mu+(1-\mu)\,\theta^s}\left[\theta^s+\frac{\mu\theta^s}{2\left(\mu+(1-\mu)\,\theta^s\right)}-\frac{\theta^s}{2}\right]\\ &= \frac{(1-\mu)}{\mu+(1-\mu)\,\theta^s}\left[\theta^s-\frac{(1-\mu)\,\theta^s}{\mu+(1-\mu)\,\theta^s}\frac{\theta^s}{2}\right]\\ &= \frac{(1-\mu)}{\mu+(1-\mu)\,\theta^s}\left(\theta^s-\tilde{\theta}^s\right).\end{aligned}$$

So, the first-order condition above can be rewritten as

$$\begin{aligned} & \frac{3}{8\beta}(1-\mu)\left(\alpha-(1-\tilde{\theta}^s)\delta\right)^2+\frac{3\delta}{4\beta}(1-\mu)\left(\alpha-(1-\tilde{\theta}^s)\delta\right)(\theta^s-\tilde{\theta}^s) \\ = \; & (1-\mu)\left[\frac{3}{8\beta}\left(\alpha-(1-\theta^s)\,\delta\right)^2-D\right] \\ \Leftrightarrow \; & \frac{3}{8\beta}\left[\left(\alpha-(1-\theta^s)\,\delta\right)^2-\left(\alpha-(1-\tilde{\theta}^s)\delta\right)^2\right]-\frac{3\delta}{4\beta}\left(\alpha-(1-\tilde{\theta}^s)\delta\right)(\theta^s-\tilde{\theta}^s)=D \\ \Leftrightarrow \; & \frac{3\delta}{4\beta}\left(\alpha-\left(1-\frac{\theta^s+\tilde{\theta}^s}{2}\right)\delta\right)(\theta^s-\tilde{\theta}^s)-\frac{3\delta}{4\beta}\left(\alpha-(1-\tilde{\theta}^s)\delta\right)(\theta^s-\tilde{\theta}^s)=D. \end{aligned}$$

This finally leads to

$$\frac{3\delta^2}{8\beta}(\theta^s-\tilde{\theta}^s)^2=D.$$

A.2 Proofs of the Propositions

In this section, I present the proofs of the propositions stated in the main text:

Proof of Proposition 1. As stated in the main text, a simple comparison of θ^* and θ^s in expressions (12) and (14) yields that $\theta^*=\theta^s=1$ when $D\geq\frac{(4\alpha-\delta)\delta}{16\beta}$ and that $0<\theta^*<\theta^s=1$ when $\frac{3\delta^2}{32\beta}\leq D<\frac{(4\alpha-\delta)\delta}{16\beta}$. It only remains to compare θ^* and θ^s for $\frac{3\delta^2}{32\beta}\leq D<\frac{(4\alpha-\delta)\delta}{16\beta}$ where $0<\theta^*<\theta^s<1$. First, rewrite θ^s as $\theta^s=\frac{2(\alpha-\delta)}{3\delta}\sqrt{\frac{24\beta D}{(\alpha-\delta)^2}}$. Then,

$$\begin{aligned} \theta^* \; < \; & \theta^s \Leftrightarrow \sqrt{1+\frac{12\beta D}{(\alpha-\delta)^2}}-1<\sqrt{\frac{24\beta D}{(\alpha-\delta)^2}} \\ \Leftrightarrow \; & 2+\frac{12\beta D}{(\alpha-\delta)^2}-2\sqrt{1+\frac{12\beta D}{(\alpha-\delta)^2}}<\frac{24\beta D}{(\alpha-\delta)^2} \\ \Leftrightarrow \; & 1-\frac{6\beta D}{(\alpha-\delta)^2}<\sqrt{1+\frac{12\beta D}{(\alpha-\delta)^2}}. \end{aligned}$$

Since $\frac{\beta D}{(\alpha-\delta)^2}>0$, the term on the left-hand side is less than 1 while the term on the right-hand side is greater than 1. Hence, for $\frac{3\delta^2}{32\beta}\leq D<\frac{(4\alpha-\delta)\delta}{16\beta}$, it is always true that $\theta^*<\theta^s$, which means that the seller engages in excessive information disclosure. ■

Proof of Proposition 2. Comparing the values of θ^* and θ^s in expressions (18) and (20), it is clear that $\theta^*=1-\frac{\alpha-2\sqrt{\beta D}}{\delta}>1-\frac{\alpha-2\sqrt{2\beta D/3}}{\delta}=\theta^s$ when $D<\frac{(\delta-\alpha)^2}{4\beta}$. For $\frac{(\delta-\alpha)^2}{4\beta}\leq D<\frac{3(\delta-\alpha)^2}{8\beta}$, since θ^* is increasing in D, it follows that $\theta^*>1-\frac{\alpha-2\sqrt{\beta D}}{\delta}$, so once again $\theta^*>\theta^s$. Next, note that

$$\frac{3\delta^2}{32\beta}\geq\frac{(4\alpha-\delta)\delta}{16\beta} \quad \Leftrightarrow \quad \alpha\leq\frac{5\delta}{8} \quad \text{(with equality when } \alpha=\frac{5\delta}{8}\text{).}$$

When $\alpha \le \frac{5\delta}{8}$, comparing θ^* and θ^s for $\frac{3(\delta-\alpha)^2}{8\beta} \le D < \frac{(4\alpha-\delta)\delta}{16\beta}$ yields

$$\begin{aligned}
\theta^* \quad &\ge \quad \theta^s \Leftrightarrow \sqrt{1+\frac{12\beta D}{(\delta-\alpha)^2}}+1 \ge \sqrt{\frac{24\beta D}{(\delta-\alpha)^2}} \\
&\Leftrightarrow \quad 2+\frac{12\beta D}{(\delta-\alpha)^2}+2\sqrt{1+\frac{12\beta D}{(\delta-\alpha)^2}} \ge \frac{24\beta D}{(\delta-\alpha)^2} \\
&\Leftrightarrow \quad 1+\frac{12\beta D}{(\delta-\alpha)^2} \ge \left(\frac{6\beta D}{(\delta-\alpha)^2}-1\right)^2 \\
&\Leftrightarrow \quad \frac{24\beta D}{(\delta-\alpha)^2} \ge \left(\frac{6\beta D}{(\delta-\alpha)^2}\right)^2 \\
&\Leftrightarrow \quad D \le \frac{2(\delta-\alpha)^2}{3\beta} \quad \text{(with equality when } D = \frac{2(\delta-\alpha)^2}{3\beta}\text{)}.
\end{aligned}$$

However, it is clear by a simple comparison that

$$\frac{2(\delta-\alpha)^2}{3\beta} \ge \frac{3\delta^2}{32\beta} \ge \frac{(4\alpha-\delta)\delta}{16\beta} \quad \Leftrightarrow \quad \alpha \le \frac{5\delta}{8} \quad \text{(with equality when } \alpha = \frac{5\delta}{8}\text{)}.$$

Hence, when $\alpha \le \frac{5\delta}{8}$, it follows that $\theta^* > \theta^s$ for $\frac{3(\delta-\alpha)^2}{8\beta} \le D < \frac{(4\alpha-\delta)\delta}{16\beta}$. For $\frac{(4\alpha-\delta)\delta}{16\beta} \le D < \frac{3\delta^2}{32\beta}$, we have $\theta^s < \theta^* = 1$. And finally, for $D > \frac{3\delta^2}{32\beta}$, $\theta^s = \theta^* = 1$.

When $\frac{5\delta}{8} < \alpha \le \delta$, on the other hand, the above calculations imply that $\theta^* \gtreqless \theta^s$ *as* $D \lesseqgtr \frac{2(\alpha-\delta)^2}{3\beta}$. Hence, disclosure is insufficient if $\alpha \le \frac{5\delta}{8}$, or otherwise if $\alpha > \frac{5\delta}{8}$ and $D < \frac{2(\delta-\alpha)^2}{3\beta}$. ■

Proof of Proposition 3. Here, I compare the values of θ^* and θ^s in expressions (23) and (24). Note that

$$\frac{3\delta^2}{8\beta} \ge \frac{\alpha^2-(\alpha-\delta)^2}{4\beta} \quad \Leftrightarrow \quad \alpha \le \frac{5\delta}{4} \quad \text{(with equality when } \alpha = \frac{5\delta}{4}\text{)}.$$

Rewrite $\sqrt{\frac{8\beta D}{3\delta^2}}$ as $\frac{(\alpha-\delta)}{\delta}\sqrt{\frac{8\beta D}{3(\alpha-\delta)^2}}$. When $\alpha \le \frac{5\delta}{4}$, comparing θ^* and θ^s yields

$$\begin{aligned}
\theta^* \quad &\ge \quad \theta^s \Leftrightarrow \sqrt{1+\frac{4\beta D}{(\alpha-\delta)^2}}-1 \ge \sqrt{\frac{8\beta D}{3(\alpha-\delta)^2}} \\
&\Leftrightarrow \quad 2+\frac{4\beta D}{(\alpha-\delta)^2}-2\sqrt{1+\frac{4\beta D}{(\alpha-\delta)^2}} \ge \frac{8\beta D}{3(\alpha-\delta)^2} \\
&\Leftrightarrow \quad \left(1+\frac{2\beta D}{3(\alpha-\delta)^2}\right)^2 \ge 1+\frac{4\beta D}{(\alpha-\delta)^2} \\
&\Leftrightarrow \quad \left(\frac{2\beta D}{3(\alpha-\delta)^2}\right)^2 \ge \frac{8\beta D}{3(\alpha-\delta)^2} \\
&\Leftrightarrow \quad D \ge \frac{6(\alpha-\delta)^2}{\beta} \quad \text{(with equality when } D = \frac{6(\alpha-\delta)^2}{\beta}\text{)}.
\end{aligned}$$

It is easy to see that

$$\frac{3\delta^2}{8\beta} \geq \frac{\alpha^2 - (\alpha-\delta)^2}{4\beta} \geq \frac{6\,(\alpha-\delta)^2}{\beta} \quad \Leftrightarrow \quad \alpha \leq \frac{5\delta}{8} \quad \text{(with equality when } \alpha = \frac{5\delta}{8}\text{)}.$$

It then follows that when $\alpha \leq \frac{5\delta}{4}$, $\theta^* \gtreqless \theta^s$ *as* $D \gtreqless \frac{6(\alpha-\delta)^2}{\beta}$. When $\alpha > \frac{5\delta}{4}$, on the other hand, the above calculations imply that $\theta^* < \theta^s < 1$ for $D < \frac{3\delta^2}{8\beta}$, $\theta^* < \theta^s = 1$ for $\frac{3\delta^2}{8\beta} \leq D < \frac{\alpha^2-(\alpha-\delta)^2}{4\beta}$, and $\theta^* = \theta^s = 1$ for $D \geq \frac{\alpha^2-(\alpha-\delta)^2}{4\beta}$. Consequently, disclosure is insufficient if $\alpha \leq \frac{5\delta}{4}$ and $\frac{6(\alpha-\delta)^2}{\beta} < D \leq \frac{3\delta^2}{8\beta}$. ■

References

Board, O. (2009). Competition and disclosure. *The Journal of Industrial Economics*, 57(1):197–213.

Cheong, I. and Kim, J.-Y. (2004). Costly information disclosure in oligopoly. *The Journal of Industrial Economics*, 52(1):121–132.

Daughety, A. F. and Reinganum, J. F. (2008). Communicating quality: a unified model of disclosure and signalling. *The RAND Journal of Economics*, 39(4):973–989.

Dranove, D. and Jin, G. Z. (2010). Quality disclosure and certification: Theory and practice. *Journal of Economic Literature*, 48(4):935–63.

Dye, R. A. (1985). Disclosure of nonproprietary information. *Journal of Accounting research*, 23(1):123–145.

Fishman, M. J. and Hagerty, K. M. (1990). The optimal amount of discretion to allow in disclosure. *The Quarterly Journal of Economics*, 105(2):427–444.

Grossman, S. J. (1981). The informational role of warranties and private disclosure about product quality. *Journal of law and economics*, 24(3):461–483.

Grossman, S. J. and Hart, O. D. (1980). Disclosure laws and takeover bids. *The Journal of Finance*, 35(2):323–334.

Jovanovic, B. (1982). Truthful disclosure of information. *The Bell Journal of Economics*, pages 36–44.

Matthews, S. and Postlewaite, A. (1985). Quality testing and disclosure. *The RAND Journal of Economics*, pages 328–340.

Milgrom, P. and Roberts, J. (1986). Relying on the information of interested parties. *The RAND Journal of Economics*, pages 18–32.

Milgrom, P. R. (1981). Good news and bad news: Representation theorems and applications. *The Bell Journal of Economics*, pages 380–391.

Shin, H. S. (1994). News management and the value of firms. *The RAND Journal of Economics*, pages 58–71.